The physics of table tennis

Ricard Alert Zenón
ricard.alert.zenon@gmail.com

Institut Pere Vives Vich, Igualada
Facultat de Física de la Universitat de Barcelona

High school research project
Supervised by Ernest Fabregat Soler
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Chapter

Goals and motivation. What to do and why

Goals

- Physical description of the game

Motivation

- Combination of two of my hobbies
- Originality (today the topic has been studied in at least two other reasearch projects, at that time it was unpublished)
- Special incidence of Physics in table tennis:
  - size and mass of the ball $\Longrightarrow$ high speed (translational) and spin (rotational)
  - quantity, diversity and complexity of involved materials
  - it is said to be the fastest non-motorized sport in the world

Goals and motivation. What to do and why

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Metodology. How to do it

Reductionism

1. Detection of the basic and simple mechanisms that rule a complex physical phenomenon
2. Description of these basic mechanisms
3. Integration of the basic mechanisms to give account of the complex phenomenon

In the case of the physics of table tennis, the basic mechanisms are only two: the rebound and the spin.

Investigation method

1. Experiment or empirical fact to be explained
2. Theory (various aspects of newtonian classical mechanics)
3. Conclusions

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Section

Experiment

Procedure

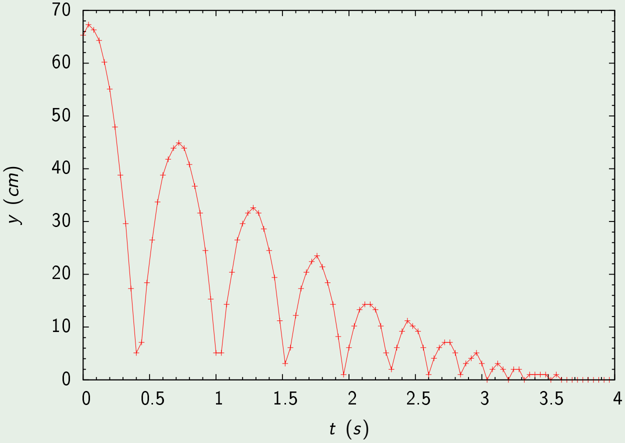
Measurement of the height of the ball versus time in consecutive free fall vertical rebounds (*Nittaku**** ball on a synthetic rubber court floor)

Instrumentation

- *MultiLog Pro* position sensor (25 *Hz*)
- *MultiLab* software

Experiment

Results



Theory

Rebound energy dissipation

- Negligible air friction (checked to be a fairly good approximation)
- Energy dissipation almost totally due to the inelastic collision
- Proposal of a simple model

$$zE_i = E_f; \qquad z \equiv \text{energy conservation factor} \tag{1}$$

$$z\left(U_i + T_i\right) = U_f + T_f \tag{2}$$

$$zmgh_i = mgh_f \Longrightarrow z = \frac{h_f}{h_i} \tag{3}$$

- Limitations of the model: a $z = z\left(E_i\right)$ dependence is observed

Theory

Restitution coefficient

Reformulation in terms of the restitution coefficient of a collision (ε)

$$\varepsilon \equiv \frac{|\vec{p}_{s,CM}|}{|\vec{p}_{e,CM}|} \qquad \text{(Newton)} \tag{4}$$

$$\text{Unidimensional ball-Earth collision} \Longrightarrow \varepsilon = \frac{v_s}{v_e} \tag{5}$$

Mechanical energy conservation in the air

$$\frac{mv_e^2}{2} = mgh_i, \quad \frac{mv_s^2}{2} = mgh_f \Longrightarrow \frac{v_e^2}{v_s^2} = \frac{h_i}{h_f} \Longrightarrow \varepsilon = \sqrt{z} \tag{6}$$

ε and z provide the same information

Conclusions

Features of the ball. Elastic model

- Greater energy loss to larger diameter (justification for the increased size of the ball to slow the game down)
  Elastic model for the ball-surface system (Hooke's law)

$$F = \frac{dp}{dt} = YS\frac{\delta L}{L} \equiv k\delta L \tag{7}$$

  $Y \equiv$Young modulus (materials' intrinsic property)
  Consider:
  - Two different size balls free falling from the same height (equal Δp)
  - Stopping time in the collision (Δt) to be independent of the size of the ball (L)
  - Contact area S to be independent of the size of the ball

Conclusions

Features of the ball. Elastic model

- Greater energy loss to larger diameter (justification for the increased size of the ball to slow the game down)

$$F = \frac{dp}{dt} \approx \frac{\Delta p}{\Delta t} \quad \text{independent of the size} \tag{8}$$

$$\frac{\delta L_{\text{large}}}{L_{\text{large}}} = \frac{\delta L_{\text{small}}}{L_{\text{small}}} \Longrightarrow \delta L_{\text{large}} > \delta L_{\text{large}} \tag{9}$$

$$E_e = E_i - E_f = mg\left(h_i - h_f\right) = \frac{k\left(\delta L\right)^2}{2} = \frac{F}{2}\delta L \tag{10}$$

$$E_{e,\text{large}} > E_{e,\text{small}} \tag{11}$$

Conclusions

Features of the racket. Elastic model

- Greater energy loss to thicker sponge: increased L
- Less energy loss using the former speed glues: increased Y
- Greater energy loss to larger flexibility of the rubber: reduced Y (systems such as *Butterfly*'s *High Tension* attempt to minimize this effect)
- Greater control to larger flexibility of the rubber and/or thicker sponge: due to the greater deformation δL

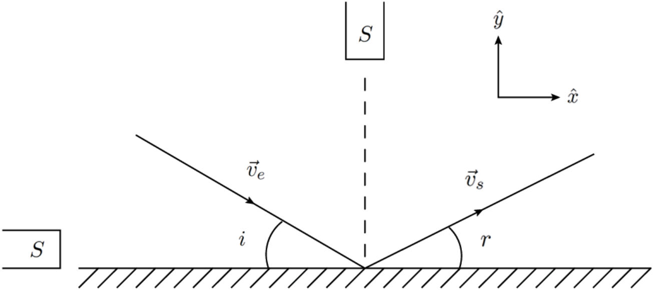
Balance between the various aspects is needed

Section

Experiment

Procedure

Measurement of the ball velocity before and after the collision assuming uniform rectilinear motion during the tenth of a second previous and next to the collision



Experiment

Instrumentation

- *MultiLog Pro* position sensor
- *MultiLab* software

Results

$$v_{e,x} = 0.21 \pm 0.07\, m/s, \qquad v_{e,y} = -1.63 \pm 0.07\, m/s \tag{12}$$

$$v_{s,x} = 0.20 \pm 0.07\, m/s, \qquad v_{s,y} = 0.92 \pm 0.07\, m/s \tag{13}$$

$$\varepsilon = 0.57 \pm 0.04 \tag{14}$$

$$i = \arctan \frac{|v_{e,y}|}{|v_{e,x}|} = 83 \pm 2^{\circ}, \qquad r = \arctan \frac{|v_{s,y}|}{|v_{s,x}|} = 78 \pm 4^{\circ} \tag{15}$$

Theory

Incidence and reflection angles

Reflection and incidence angles do not match ($r \neq i$) due to collision inelasticity

The experimentally encountered relation $v_{e,x} \approx v_{s,x}$ points towards finding r from ε for a given i

$$\varepsilon = \frac{|\vec{v}_s|}{|\vec{v}_e|} = \frac{\sqrt{v_{s,x}^2 + v_{s,y}^2}}{\sqrt{v_{e,x}^2 + v_{e,y}^2}} = \frac{|v_{s,x}|\sqrt{1+\tan^2 r}}{|v_{e,x}|\sqrt{1+\tan^2 i}} \approx \sqrt{\frac{1+\tan^2 r}{1+\tan^2 i}} \tag{16}$$

$$r \approx \arctan\sqrt{\varepsilon^2\left(1+\tan^2 i\right)-1}\,; \qquad \varepsilon > \frac{1}{\sqrt{1+\tan^2 i}} \tag{17}$$

It is worth pointing out that $r\left(\varepsilon = 1\right) = i$ as expected

Conclusions

Incidence and reflection angles

- Energy dissipation at the ball-surface contact produces a loss in the reflection angle (a horizontal, without gravity effects, and spinless situation has been studied)
- The main dissipation source in a collision between a moving ball and a resting racket is the perpendicular elastic deformation, not the tangential friction
- The reflection angle can be determined from the incidence angle once the ball-surface system restitution coefficient is known

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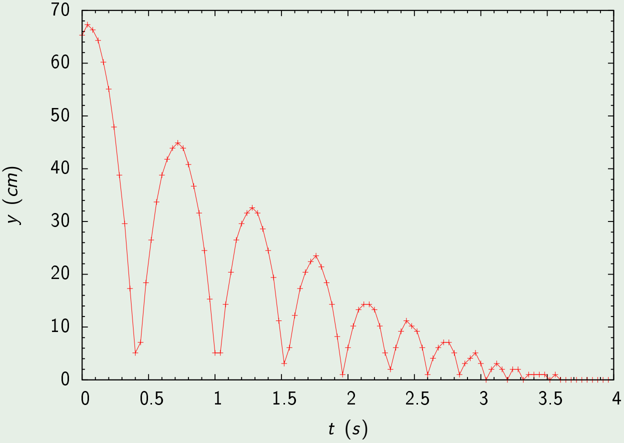


Theory

Height of the ball versus time $y(t)$

Free fall with $y_{0,n} = 0$

$$y(t) = v_n (t - t_{n-1}) - \frac{g (t - t_{n-1})^2}{2} \equiv v_n \tilde{t}_n - \frac{g \tilde{t}_n^2}{2} \qquad (18)$$

$y_{0,n}$ ≡initial position of the nth rebound
v_n ≡initial velocity of the nth rebound
t_n ≡total elapsed time once the nth rebound is completed
$\tilde{t}_n$ ≡elapsed time from the beggining of the nth rebound
t_n will be calculated and functionally inverted to get $n = n(t)$

$$t_n = \sum_{k=0}^{n} T_k; \qquad T_k \equiv \text{duration of the } k\text{th rebound} \qquad (19)$$

Theory

Height of the ball versus time $y(t)$

Consider ε to be rebound independent, that is, energy independent

$$T_k = 2\sqrt{\frac{2h_k}{g}} = 2\varepsilon^k \sqrt{\frac{2h_0}{g}} \equiv T_0 \varepsilon^k \qquad (20)$$

Geometric progression sum

$$t_n = \sum_{k=0}^{n} T_k = T_0 \frac{1-\varepsilon^{n+1}}{1-\varepsilon} \Longrightarrow t_{n-1} = T_0 \frac{1-\varepsilon^n}{1-\varepsilon} \qquad (21)$$

Theory

Height of the ball versus time $y(t)$

Invert the function in order to get $n(t)$

$$n = \frac{\ln\left(1 - (1-\varepsilon)\frac{t_{n-1}}{T_0}\right)}{\ln \varepsilon} \tag{22}$$

When replacing $t_{n-1} \to t$ only an integer part function (floor function) needs to be added

$$n = \left\lfloor \frac{\ln\left(1 - (1-\varepsilon)\frac{t}{T_0}\right)}{\ln \varepsilon} \right\rfloor \tag{23}$$

Theory

Height of the ball versus time $y(t)$

Initial velocities are

$$v_n = \varepsilon^n \sqrt{\frac{gh_0}{2}} \tag{24}$$

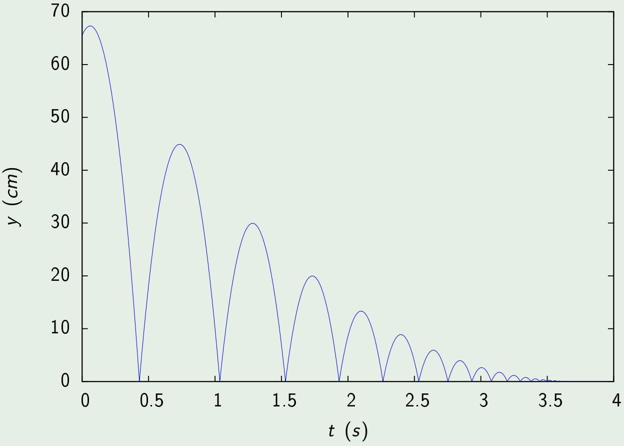
Finally

$$y(t) = \sqrt{\frac{gh_0}{2}} \varepsilon^n \tilde{t}_n - \frac{g\tilde{t}_n^2}{2} \tag{25}$$

- An analytical expression for the height of the ball at any time has been obtained taking into account the rebounds
- Only knowlegde of the initial height h_0, gravity g and restitution coefficient ε is demanded

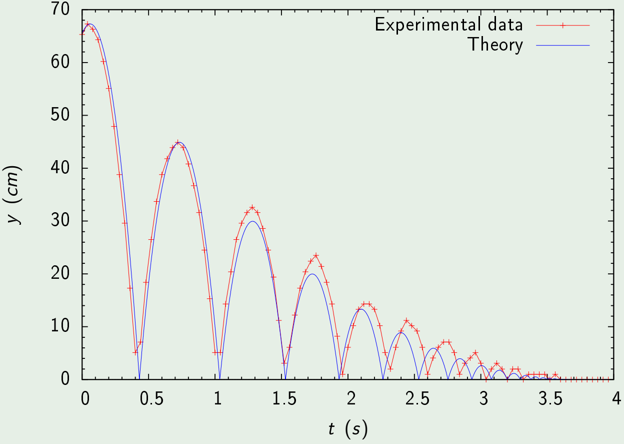
Conclusions

Theoretical graph of the height $y(t)$



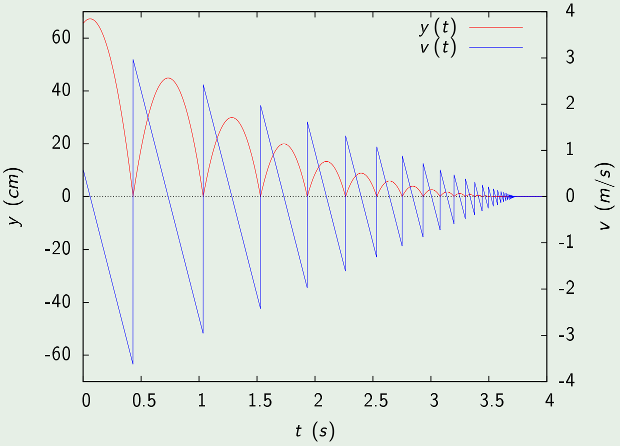
Conclusions

Theory-experiment comparison of the height $y(t)$



Conclusions

Theoretical graphs of the height $y(t)$ and the velocity $v(t)$



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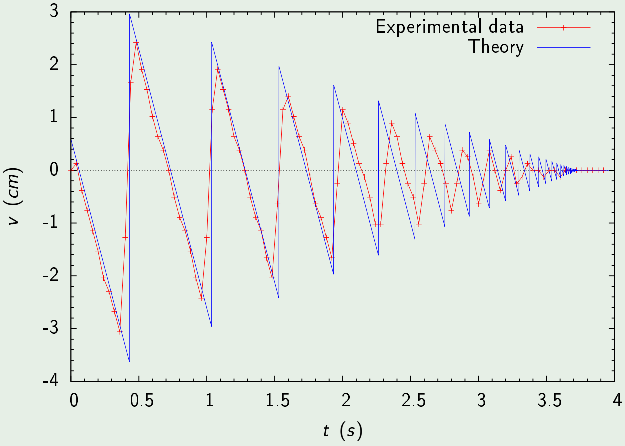
Theory-experiment comparison of the velocity $v(t)$



Chapter

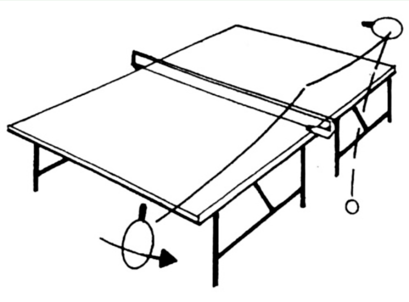
Section

Experiment

Empirical evidence

Spinning balls rebound in a different way than the non-spinning ones

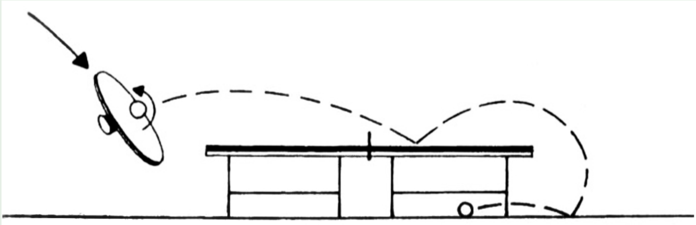
- Sidespin: a sidespinning ball rebounds into a different plain from that of incidence (it moves sideways)



Experiment

Empirical evidence

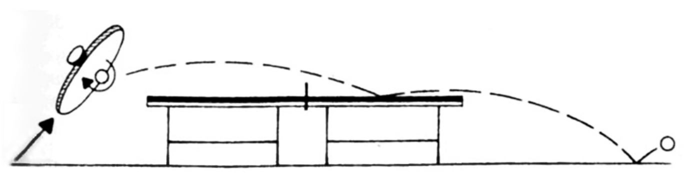
- Backspin: a backspinning ball rebounds at a greater reflection angle than a non-spinning one (it goes upwards when it contacts the table until it eventually goes backwards and it goes downwards when it contacts the racket)



Experiment

Empirical evidence

- Topspin: a topspinning ball rebounds at a lower reflection angle than a non-spinning one (it goes downwards when it contacts the table and it goes upwards when it contacts the racket)

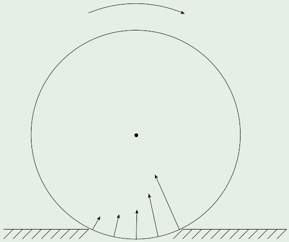


Theory

Rolling model

Consider a spinning ball free falling towards a flat surface

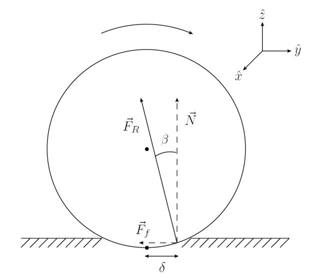
- At the time of contact, the spin creates an asymmetry in the pressure that receives the contact region of the ball. The origin of this asymmetry is the viscoelastic response of materials, which deform faster than they recuperate their initial shape (elastic hysteresis)



Theory

Rolling model

- The resultant force $\vec{F}_R$ is offset δ from the central point of the contact $\Longrightarrow$ it creates a torque $\vec{M}_{\text{rel}}$



$$\vec{M}_{\text{rel}} = \vec{r} \times \vec{F}_R \approx \delta F_R \sin\left(\frac{\pi}{2} + \beta\right) \hat{x} =$$

$$= \delta F_R \cos\beta \hat{x} = \delta N \hat{x} \quad (26)$$

- The tangential component of $\vec{F}_R$ creates a small friction

$$\vec{F}_f = -F_R \sin\beta \hat{y} = \frac{N}{\cos\beta} \sin\beta \hat{y} = -N \tan\beta \hat{y} \equiv -\mu N \hat{y} \quad (27)$$

Theory

Rolling model

- A torque $\vec{M}$ changes the angular momentum $\vec{L}$

$$\vec{M} = \frac{d\vec{L}}{dt} \tag{28}$$

Separation of the angular momentum

$$\vec{L} = \sum_{i=1}^{N} \vec{r}_i \times \vec{p}_i = \sum_{i=1}^{N} \left(\vec{r}_{CM} + \vec{r}_i\,'\right) \times m_i \left(\vec{v}_{CM} + \vec{v}_i\,'\right) =$$

$$= \sum_{i=1}^{N} \vec{r}_i\,' \times \vec{p}_i\,' + \vec{r}_{CM} \times \vec{p}_{CM} \equiv \vec{L}_{\text{rel}} + \vec{L}_{CM} \tag{29}$$

Theory

Rolling model

- The torque $\vec{M}_{\text{rel}}$ modifies the angular momentum $\vec{L}_{\text{rel}}$ (it is internal)

$$\vec{L}^f_{\text{rel}} \approx \vec{L}^i_{\text{rel}} + \tau \vec{M}_{\text{rel}} \tag{30}$$

$\tau \equiv$contact duration
Moment of inertia I of the ball (spherical shell)

$$\vec{L}^i_{\text{rel}} = I\vec{\omega}_i = -\frac{2mR^2}{3}\omega_i \hat{x} \tag{31}$$

$$\vec{L}^f_{\text{rel}} \approx -\frac{2mR^2}{3}\omega_i \hat{x} + \tau\delta N \hat{x} \equiv -\frac{2mR^2}{3}\omega_f \hat{x} \tag{32}$$

Theory

Rolling model

- The ball rebounds at a lower spin

$$\omega_f \approx \omega_i - \frac{3\tau\delta N}{2mR^2} \tag{33}$$

- Total angular momentum is approximately conserved

$$\vec{L}^f = \vec{L}^f_{\text{rel}} + \vec{L}^f_{\text{CM}} \approx \vec{L}^i = \vec{L}^i_{\text{rel}} \tag{34}$$

- When rebounding, the ball acquires an orbital angular momentum that makes it deviate towards the empirically observed direction

$$\vec{L}^f_{\text{CM}} \approx -\tau\delta N\hat{x} \tag{35}$$

Conclusions

Features of the ball

- Less spin to larger diameter (justification for the increased size of the ball to slow the game down)
  If the ball is tangentially contacted with a force $\vec{F} = F_t\hat{\theta}$ for a period of time τ (cylindrical coordinates of the ball)

$$\vec{M} = \vec{r} \times \vec{F} = \frac{d\vec{L}}{dt} \Longrightarrow \vec{L} = \int_0^\tau RF_t\hat{z}dt = \tau RF_t\hat{z} \qquad (36)$$

  Moment of inertia I of the ball (spherical shell)

$$\vec{L} = \frac{2mR^2}{3}\vec{\omega} \Longrightarrow \vec{\omega} = \frac{3\tau F_t}{2mR}\hat{z} \qquad (37)$$

Conclusions

Features of the racket

- Greater spin to larger flexibility of the rubber and/or thicker sponge: increased rolling coefficient δ
- Action mechanism of pimpled rubbers:
  - decreased contact area $\Longrightarrow$ less grip $\Longrightarrow$ less spin (more pronounced to lower pimple area)
  - pimples asymmetric deformation, squashed at the attacking flank but bended at the other. Elastic restitution of this deformation can reverse the spin of the ball (more pronounced to greater pimple length)

Section

Experiment

Procedure

- Direct measurement of the deviation angle of a spinning ball suspended on a pendulum in an air flow (trigonometry and scaling on a photography), φ_d
- Measurement of the deviation angle of a spinning ball suspended on a pendulum in an air flow according to Magnus theory, φ_t
  - Measurement of the air speed (anemometer), u
  - Measurement of the rotation speed of the ball (video), v

Instrumentation

- Rope, nylon thread, clothes peg
- Fan
- Photography and video camera

Experiment

Results

- Direct measurement

$$\varphi_d = \arcsin \frac{x + R}{a} = 3.4 \pm 0.2^o \quad (38)$$

$x \equiv$horizontal separation of the pendulum
$a \equiv$pendulum length

- Indirect measurement

$$u = 2.5 \pm 0.1 \; m/s \quad (39)$$

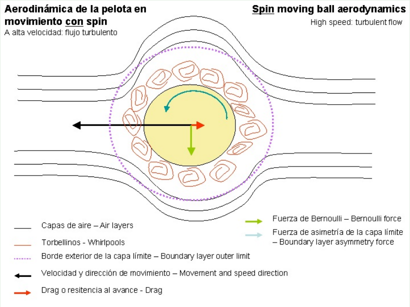
$$\nu = 2.00 \pm 0.06 \; Hz \quad (40)$$

Loading

Theory

Magnus effect

Kutta-Joukowsky theorem: solution of the problem of an arbitrarily shaped body in a bidimensional ideal, stationary, incompressible flow $\vec{u}$ with vorticity $\vec{\Gamma}$ (it remains potential or irrotational)



$$\vec{F}_M = \rho \vec{u} \times \vec{\Gamma} \qquad \text{Magnus force on the body} \tag{41}$$

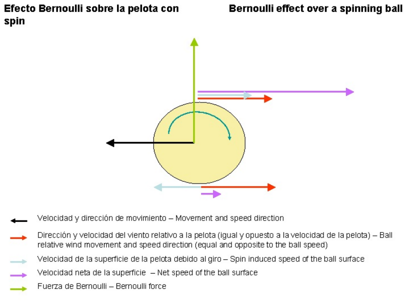
$\rho \equiv$fluid density

Theory

Magnus effect

Physical interpretation:
stick in the fluid-solid interface
$\Longrightarrow$ speed difference
$\Longrightarrow$ pressure difference P
$\Longrightarrow$ force
Bernoulli's theorem

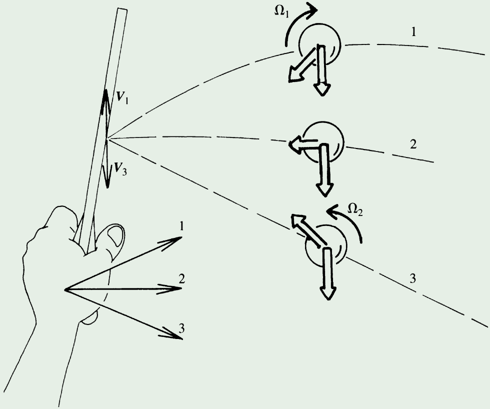
$$\vec{\nabla}\left(P+\frac{\rho u^2}{2}\right)=\vec{0} \quad (42)$$



- Boundary layer turbulence $\Longrightarrow$ slight interfacial slip $\Longrightarrow$ decreased Magnus effect and aerodynamic drag (golf balls)

Conclusions

Spinning balls trajectory



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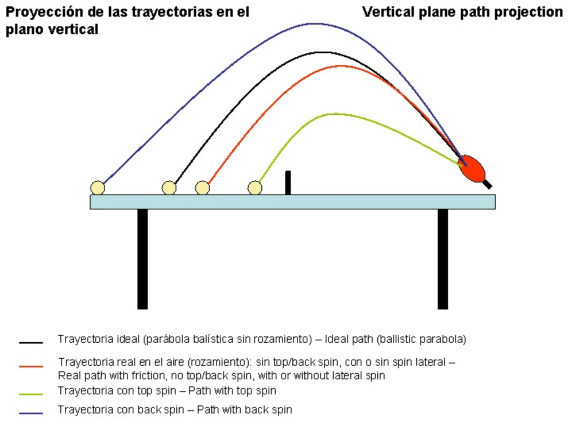

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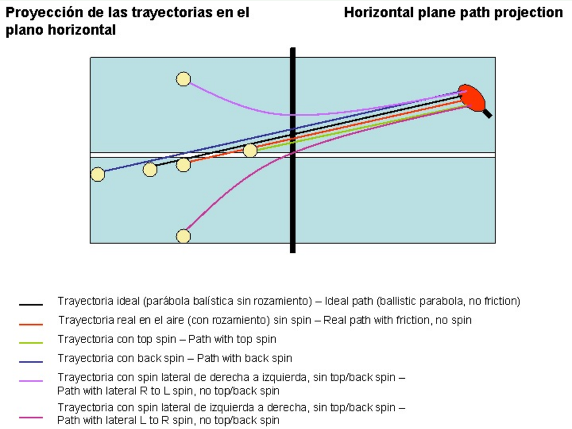

Conclusions

Theory-experiment comparison

$$\text{Stick} \qquad v_+ = u + v, \quad v_- = u - v; \quad v = \omega R = 2\pi\nu R \tag{43}$$

$$\text{Bernoulli's theorem} \qquad \Delta P = \frac{\rho}{2}\left(v_+^2 - v_-^2\right) = 2\rho u v \tag{44}$$

$$\text{Magnus force} \qquad F_M = S\Delta P = \pi R^2 \Delta P = 2\pi\rho u v R^2 \tag{45}$$

$$\text{Pendulum dynamics} \qquad \varphi_t = \arctan\frac{F_M}{mg} = 4.3 \pm 0.6^{\circ} \tag{46}$$

Consistent result with $\varphi_d = 3.4 \pm 0.2^{\circ}$

- Magnus theory describes reasonably well, even in quantitative terms, the deviation of aerodynamic origin of spinning balls

Chapter

General conclusions

Goals achievement

A description of the basic physical mechanisms ruling the various phenomena playing a role in table tennis rallies has been achieved

- The rebound
  - Energy loss in the rebound and its relation to the main features of the ball and the racket
  - Reflection angle loss in the rebound
  - Kinematical characterization of a series of vertical rebounds
- The spin
  - Trajectories of rebounding spinning balls and their relation to the main features of the ball and the racket
  - Trajectories of spinning balls moving through air

Further work

La màgia dels efectes (The magic of spin)

Author: Albert Martínez Vall
High school: Col·legi Sant Miquel dels Sants, Vic
Academic year: 2009
Supervisor: Miquel Padilla Muñoz

- Only the spin issue is studied but to a greater depth in the experimental ground
- Bidimensional rebound experiments similar to ours but using back and topspinning balls are performed and compared
- Simulations on ball aerodynamics are performed

Further work

La física i el tennis taula (Physics and table tennis)

Author: Oriol Abante Alert
High school: Col·legi La Mercè, Martorell
Academic year: 2011
Supervisor: Josep Anton Garrido Alarcón

- Generic study mainly focused on the sportive side (table tennis materials and technique)
- It contains a section concerning table tennis biomechanics
- Experiments on rubbers tensile resistance are performed
- A survey and two interviews are included

Further research suggestions

Some issues likely to be addressed

- Improvement of accuracy and amount of all of the experiments
- Deeper study of the elastic properties of materials
- Experimental quantitative verification of the rolling model
- Detailed study of the action mechanisms of pimpled rubbers
- Deeper study of ball aerodynamics including turbulence effects

Thank you very much!